REFERENCE MATHEMATICAL MODULE

WHAT IS MATHEMATICAL MODELLING?

Mathematical modeling is the use of mathematics to

- describe real-world phenomena
- investigate important questions about the observed world
- explain real-world phenomena
- test ideas
- make predictions about the real world

The real world refers to

- engineering
- physics
- physiology
- ecology
- wildlife management
- chemistry
- economics
- sports . . .

Experimental scientists are very good at taking apart the real world and studying small components. Since the real world is nonlinear, fitting the components together is a much harder puzzle.

Mathematical modeling allows us to do just that. Ideally, the combination of science and modeling leads to a complete understanding of the phenomenon being studied.

How To Build a Mathematical Model?

1] Identify All Decision Variables

Controllable parameters whose values can be controlled by decision maker, which affect functioning of system. Denote them by x1, . . . , xn.

$$x = \text{Decision vector} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = (x_1, \ldots, x_n)^T$$

2] Identify Objective Function and All Constraints on Decision Variables

$g_i(x) = b_i$ ------- An Equality Constraint

$g_i(x) \geq b_i$ ------- Inequality Constraints

$g_i(x) \leq b_i$ ------- Inequality Constraints

Constraint Functions, Right Hand Side (RHS) Constants.

$x_j \geq b_j$ or $x_j \leq b_j$ Lower or Upper bound constraints on individual variables.

$x_j \geq 0$ Lower bound constraint called Nonnegativity restriction.

Objective function called is --

a. COST FUNCTION if to be min.
b. PROFIT FUNCTION if to be max.

Some Definitions:

LINEAR FUNCTION:

One of form c1x1 + . . . + cnxn
Where c = (c1, . . . , cn) is coefficient vector of variables in it.

Example: $x \in R^4$. $3x_2 - 7x_4$ is a linear function with coefficient vector (0, 3, 0,−7).

AFFINE FUNCTION:

A linear function + a constant, i.e., One of form c0 + cx.

FEASIBLE SOLUTION:

A vector x that satisfies all the constraints.

OPTIMUM SOLUTION:

A feasible solution that gives the best value for objective function among all feasible solutions.

LINEAR PROGRAM:

Optimization problem in which objective function and all constraint functions are linear.

Steps in Modeling A Linear Program

1. LIST ALL DECISION VARIABLES:
   Each decision variable is the level at which an ACTIVITY is carried out.

2. VERIFY LINEARITY ASSUMPTIONS:
   Proportionality Assumption and Additivity Assumption.

Must hold for objective function and all constraint functions.

3. VERIFY ALL VARIABLES ARE CONTINUOUS VARIABLES:

4. CONSTRUCT OBJECTIVE FUNCTION:

5. IDENTIFY ALL CONSTRAINTS & BOUNDS ON INDIVIDUAL VARIABLES:

   Each constraint is Material Balance Equation or Inequality of an ITEM.

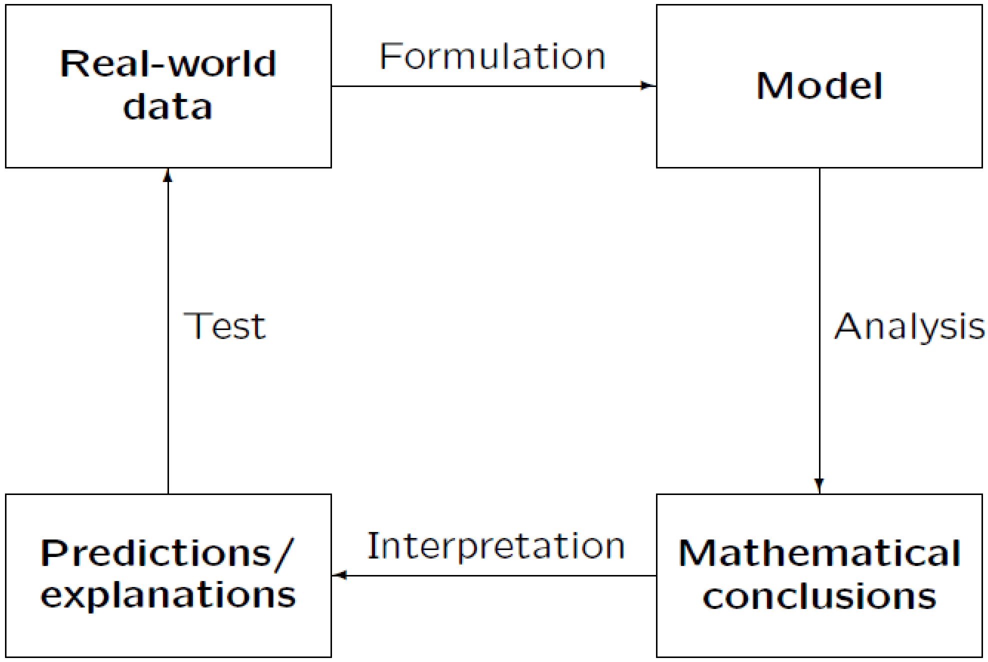
EXAMPLES of real-world questions that can be investigated with mathematical models

- Suppose there is a baseball strike. We might be interested in predicting the effects of higher players' salaries on the long-term health of the baseball industry.

- In the management of a fishery, it may be important to determine the optimal sustainable yield of a harvest and the sensitivity of the species to population fluctuations caused by harvesting.

One can think of mathematical modelling as an activity or process that allows a mathematician to be a chemist, an ecologist, an economist, a physiologist . . . .
Instead of undertaking experiments in the real world, a modeller undertakes experiments on mathematical representations of the real world.

Process of mathematical modelling



There is no best model, only better models.

Challenge in mathematical modelling

". . . not to produce the most comprehensive descriptive model
but
to produce the simplest possible model that incorporates the major features of the phenomenon of interest."

Howard Emmons

Two hands-on modelling activities –

- For the first feature, we built a model considering the user's experience, interest and knowledge background to provide search results based on individual needs.
- For the second one, we recommend resources by collaborative fIltering according to the other users' evaluation to the search results.

Consider the following situations:

Situation 1:

Users may have different intentions for the same query, e.g., searching for "jaguar" by a car fan has a completely different meaning from searching by an animal specialist.

The probability that user may find links related to car will be - P(A) = (n-r)!/n!
i.e may be half or less or more. And that of animal will be P(B) = 1- P(A).

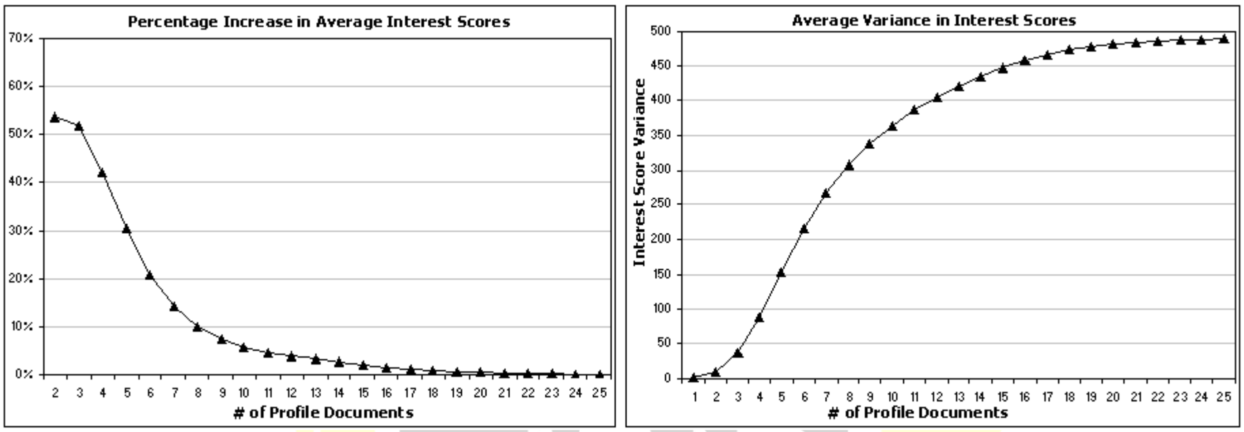
Situation 2:

A historian may enter the query *Madonna and child* while browsing Web pages about art history, while a music fan may issue the same query to look for updates on the famous pop star.

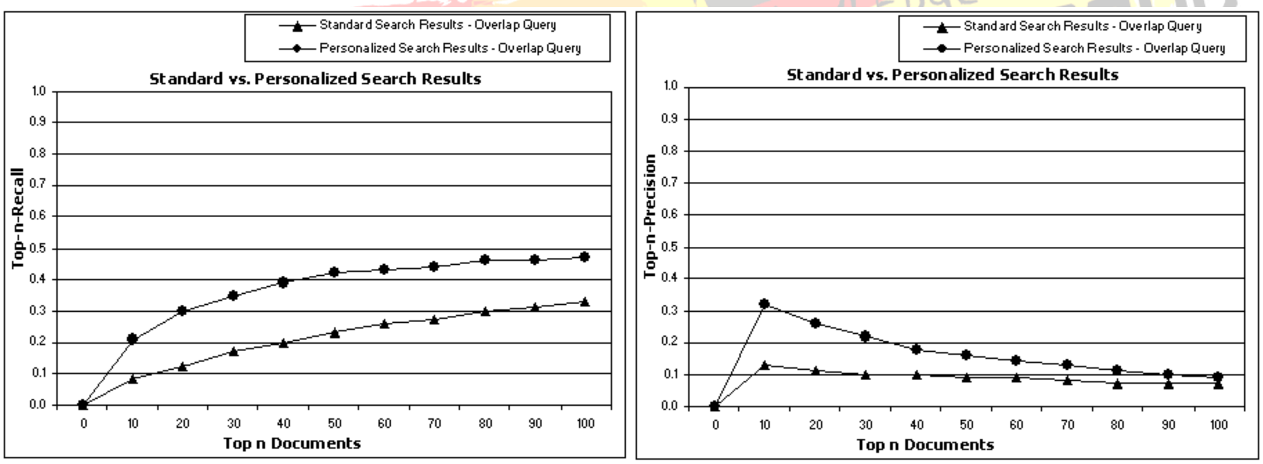
Situation 2:

Two users searching for the term 'jazz' may have different perspective, however, search engines are not able to differentiate the meaning of the word in terms of the users need. As newer topics of discussions emerge on the web and the vocabulary used is at times beyond our ken, it becomes difficult to define "the user need" to the search engine.

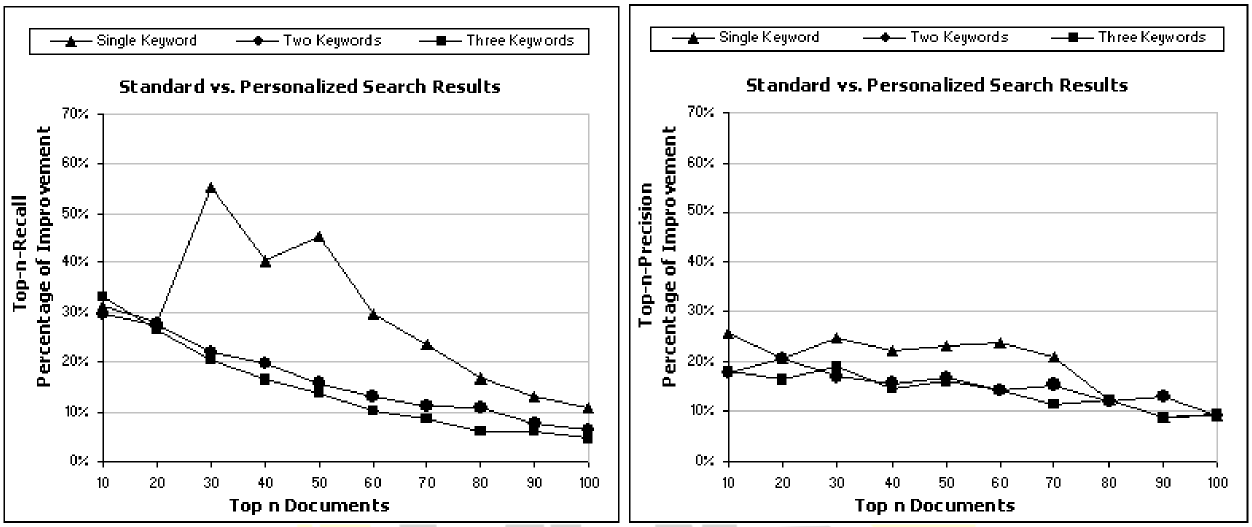
According to search of a user we may get the observations as follows.



The average rate of increase and average variance in *Interest Scores* as a result of incremental updates.



Average *Top-n Recall* and *Top-n Precision* comparisons between the personalized hybrid search and standard search using "overlap queries".



Percentage of improvement in *Top-n Recall* and *Top-n Precision* achieved by personalized hybridsearch relative to stand & search with various query sizes.

Experimental Effects:

TABLE SUMMARY OF THE EXPERIMENTAL RESULTS

Query	best	worst	average	original	improving
topic-map	0.7770	0.4691	0.7156	0.5010	55.10%
entropy	0.7061	0.3788	0.6277	0.8915	-20.79%
memory	0.5947	0.2501	0.5222	0.7138	-16.69%
SVM	0.7773	0.4989	0.7170	0.7882	-1.38%
LDA	0.8229	0.3197	0.7314	0.6799	21.03%
KMP	0.6302	0.2137	0.5545	0.4779	31.87%
Dijkstra	0.7891	0.3545	0.7101	0.7425	6.28%
visualization	0.5111	0.1958	0.4319	0.7246	-29.46%
neural-networks	0.9438	0.3339	0.7922	0.9400	0.40%
machine-learning	0.9680	0.3394	0.8537	0.8951	8.15%
Avg	0.7788	0.3509	0.6919	0.7366	2.26%
standard deviation	0.1456	0.0993	0.1299	0.1562	0.2435

According to search the RESULTS graphs will be as follows :

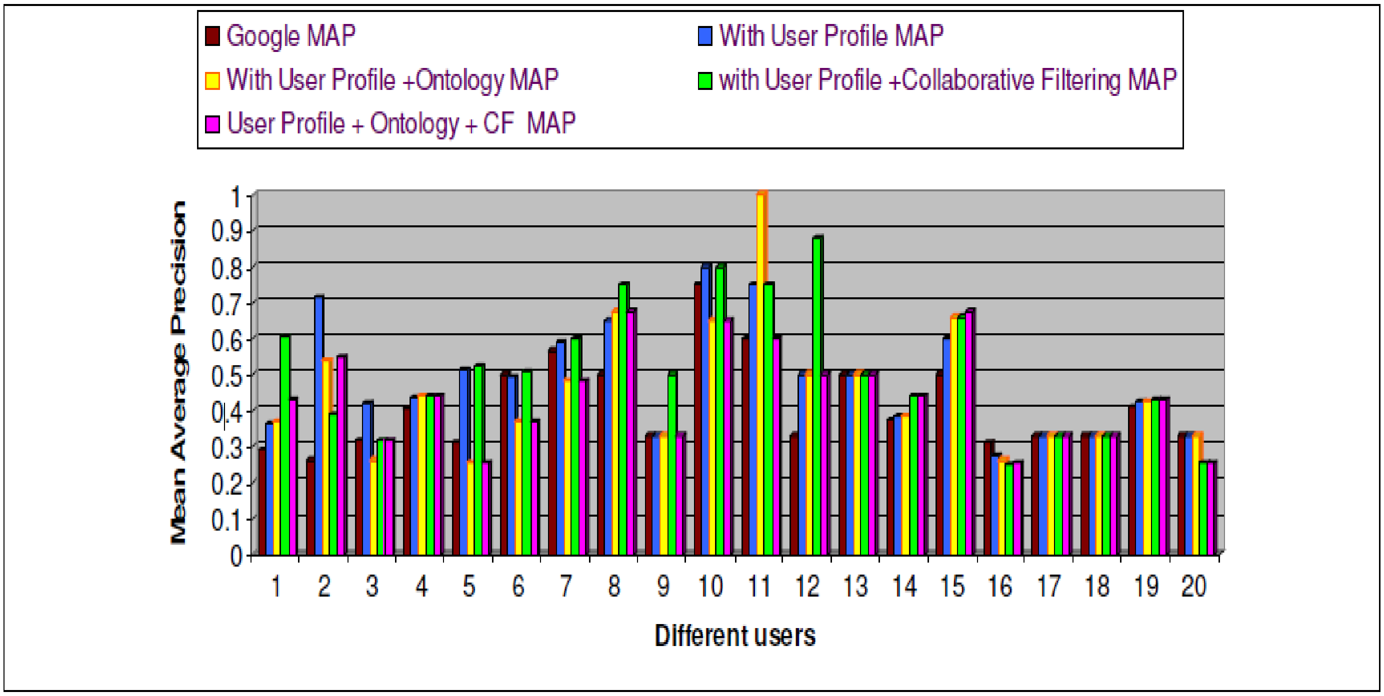


Figure : Graph showing the Mean Average Precision for Google and Proposed approach

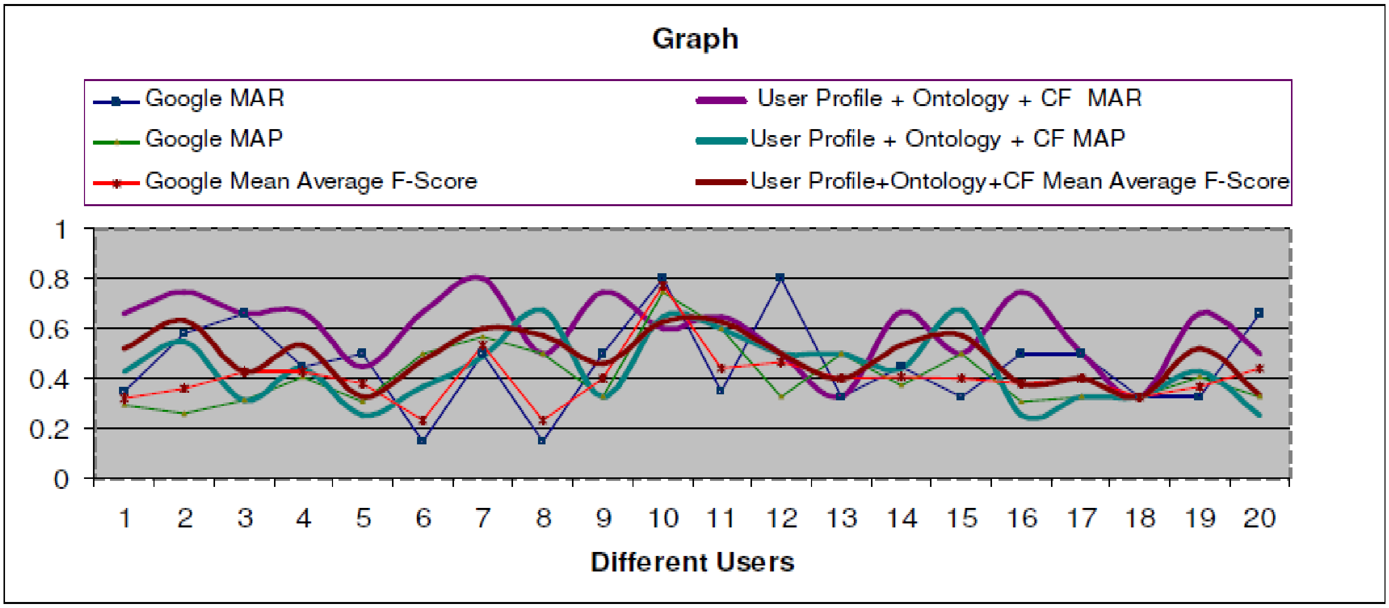


Figure : Graph showing Precision/ Recall/ F-Score for Google (Phase I) Vs Proposed Approach (Phase II)